Model partitioning and time integration

NREL/DOE Workshop on the New Modularization Framework for the FAST Wind Turbine CAE Tool

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Talk goals & outline

Talk goals: Describe partitioning schemes, module-coupling methods, and time integration for the new modularized FAST framework

Talk outline:

- Project goals
- Definitions
- Loose vs. tight coupling
- Fast as a glue code
- Example system
- Preliminary results
- Future work

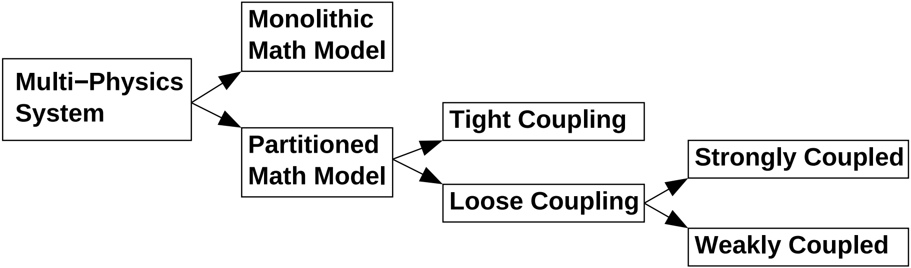
Project goals

Within the context of the new FAST modularization [1], provide FAST Module Developers with guidance on

- effective partitioning
- partition coupling
- time integration
- tight vs. loose coupling

Multi-physics modeling: Taxonomy

- Vocabulary/taxonomy surrounding multi-physics modeling and simulation is varied and sometimes contradictory
- Here, we use the following taxonomy (see Refs. [2, 3]):



Multi-physics modeling: Monolithic vs. partitioned

Monolithic math model:

- Single eqn. set that is inherently “tightly” coupled
- Different “systems” share degrees of freedom at spatial interfaces (e.g., fluid-structure interface)
- Requires a single time integrator and matching spatial and temporal meshes

Partitioned math model:

- Each partition can be time integrated separately
- Allows great flexibility in modeling
- Allows for non-matching spatial and temporal meshes
- Coupling partitioned models may introduce accuracy and/or numerical-stability issues

Coupling model partitions: Tight vs. loose

Tight coupling:

- Partitioned-model equations are assembled into a single system; single time integrator
- Matching temporal meshes; may have non-matching spatial meshes
- Likely requires differential-algebraic-equation (DAE) solver
- Allows for linearized analyses (time and/or modal)

Loose coupling:

- Partitioned-model equations are time integrated in a *conventional serial staggered* procedure [3]
- Different time-integrators can be used for different partitions
- Allows for non-matching temporal and spatial meshes

Loose coupling: Weak vs. strong

Weak vs. strong coupling is associated with data sharing between partitions during time integration

Weak loose coupling:

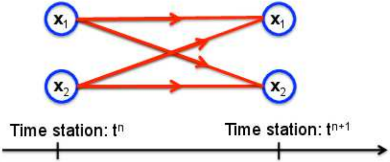
- Each partition is advanced from t to $t + \Delta t$ using other-partition interface data **only** at t
- Also known as **explicit** coupling

Strong loose coupling:

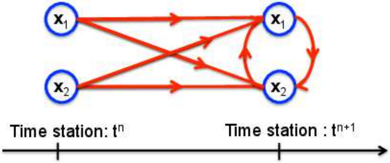
- Each partition is advanced from t to $t + \Delta t$ using other-partition interface data at $t + \Delta t$ and possibly t
- Also known as **implicit** coupling

Weak (explicit) vs. strong (implicit) staggered coupling: Schematics

- Consider staggered integration of two partitions:



(a) Weak Coupling

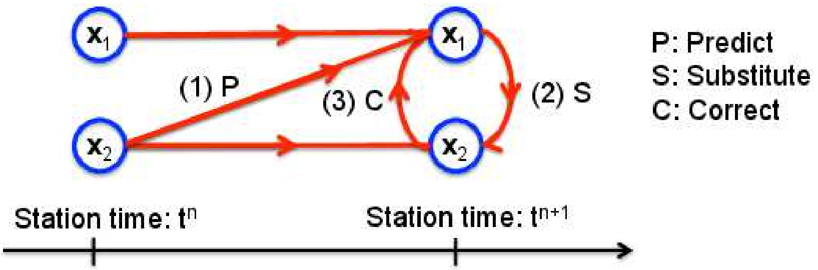


(b) Strong Coupling

- Because partitions are updated sequentially, **direct** solution of fully implicit coupling is not feasible

Strong (implicit) coupling via predictor-corrector coupling

Solution required for time advancement in implicit coupling can be solved **iteratively** through a predictor-corrector approach



FAST as a glue code

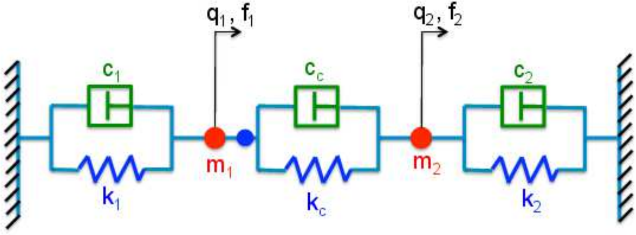
- FAST will function as a **glue code** for coupling modules/partitions [1]
- The underlying model for each module/partition will be a **state-space representation**:

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{X}\left(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t)\right) \\
\mathbf{y}(t) &= \mathbf{Y}\left(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t)\right) \\
0 &= \mathbf{Z}\left(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t)\right)
\end{aligned}
$$

  where **x** is the state, **y** is the system output, **u** is the system input, and **z** is the constraint
- For time-dependent partial-differential eqs., this is a **method of lines** approach; spatial derivatives have been discretized
- Numerical time integration depends on choice of **tight** versus **loose** coupling of modules

Example monolithic system

Two-degree-of-freedom damped linear oscillator, with applied forces $f_1(t)$, $f_2(t)$:

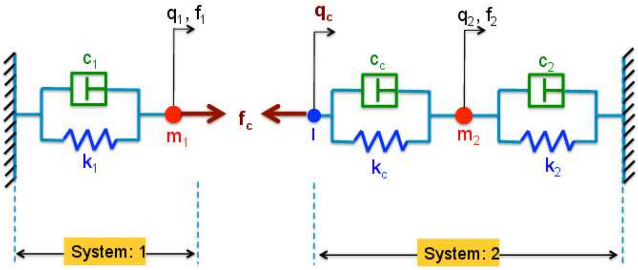


$$\mathbf{x} = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T, \quad \mathbf{y} = \varnothing, \quad \mathbf{u} = \varnothing, \quad \mathbf{z} = \varnothing$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_c+k_1}{m_1} & -\frac{c_c+c_1}{m_1} & \frac{k_c}{m_1} & \frac{c_c}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_c}{m_2} & \frac{c_c}{m_2} & -\frac{k_c+k_2}{m_2} & -\frac{c_c+c_2}{m_2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{f_1}{m_1} \\ 0 \\ \frac{f_2}{m_2} \end{bmatrix}$$

Example partitioned system

Example partitioning:



- Required introduction of coupling force f_c, which functions as a **Lagrange multiplier**

Partitioning

System 1:

$$\mathbf{x}_1 = [q_1, \dot{q}_1]^T, \quad \mathbf{y}_1 = [q_1, \dot{q}_1]^T, \quad \mathbf{u}_1 = [f_c], \quad \mathbf{z}_1 = \varnothing$$

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} \end{bmatrix} \mathbf{x}_1 + \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} \mathbf{u}_1 + \begin{bmatrix} 0 \\ \frac{f_1}{m_1} \end{bmatrix} \\ &= \mathbf{A}_1 \mathbf{x}_1 + \mathbf{B}_1 \mathbf{u}_1 + \mathbf{f}_1 \end{aligned}$$

System 2:

$$\mathbf{x}_2 = [q_2, \dot{q}_2]^T, \quad \mathbf{y}_2 = [c_c(\dot{q}_2 - \dot{q}_c) + k_c(q_2 - q_c)]$$

$$\mathbf{u}_2 = [q_c, \dot{q}_c]^T, \quad \mathbf{z}_2 = \varnothing$$

$$\begin{aligned} \dot{\mathbf{x}}_2 &= \begin{bmatrix} 0 & 1 \\ -\frac{k_c+k_2}{m_2} & -\frac{c_c+c_2}{m_2} \end{bmatrix} \mathbf{x}_2 + \begin{bmatrix} 0 & 1 \\ \frac{k_c}{m_2} & \frac{c_c}{m_2} \end{bmatrix} \mathbf{u}_2 + \begin{bmatrix} 0 \\ \frac{f_2}{m_2} \end{bmatrix} \\ &= \mathbf{A}_2 \mathbf{x}_2 + \mathbf{B}_2 \mathbf{u}_2 + \mathbf{f}_2 \end{aligned}$$

Partition input-output relationships

Input-output relationships: $0 = \mathbf{u}_1 - \mathbf{y}_2$, $0 = \mathbf{u}_2 - \mathbf{y}_1$

Given

$$\mathbf{y}_1 = [q_1, \dot{q}_1]^T, \quad \mathbf{u}_1 = [f_c],$$

$$\mathbf{y}_2 = [c_c(\dot{q}_2 - \dot{q}_c) + k_c(q_2 - q_c)], \quad \mathbf{u}_2 = [q_c, \dot{q}_c]^T$$

we find the following constraints:

$$f_c = c_c(\dot{q}_2 - \dot{q}_c) + k_c(q_2 - q_c)$$
$$q_1 = q_c$$
$$\dot{q}_1 = \dot{q}_c$$

Partitioned system: Tight coupling

- Partitions can be assembled directly into a global system:

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} \\ 0 & \mathbf{B}_2 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix}$$

$$\mathbf{0} = \begin{bmatrix} \mathbf{u}_1 - \mathbf{y}_2 \\ \mathbf{u}_2 - \mathbf{y}_1 \end{bmatrix} \Rightarrow \mathbf{0} = \begin{bmatrix} f_c - c_c\left(\dot{q}_2 - \dot{q}_c\right) - k_c\left(q_2 - q_c\right) \\ q_1 - q_c \\ \dot{q}_1 - \dot{q}_c \end{bmatrix}$$

- Treat q_c, $\dot{q}_c$, and f_c as *algebraic constraint variables*; system can be viewed as a **Differential Algebraic Equation (DAE)**, with DAE Index 1
- System can be time integrated (tightly) with standard, open-source DAE solvers, e.g. DASSL: http://www.cs.ucsb.edu/˜cse/software.html

Time integration: Loose coupling

- Each FAST module will have the capability to advance the State one time step, i.e.,

$$\dot{\mathbf{x}} = \mathbf{X}\left[t, \mathbf{x}(t), \mathbf{L}\left(\alpha, \mathbf{u}(t), \mathbf{u}(t+\Delta t)\right)\right] \xrightarrow{\text{UpdateStates}} \mathbf{x}(t+\Delta t)$$

  where **L** is a linear-interpolation operator:

$$\mathbf{L}\left(\alpha, \mathbf{u}(t), \mathbf{u}(t+\Delta t)\right) = (1-\alpha)\mathbf{u}(t) + \alpha\mathbf{u}(t+\Delta t)$$

- Input **u** is held constant while the state **x** is advanced
- “UpdateStates” embodies numerical time integration, e.g. Runge-Kutta, Adams-Bashforth-Moulton, Backwards FD
- Weak explicit coupling: $\alpha = 0$
- Strong implicit coupling: $0 < \alpha \leq 1$

Predictor-Corrector Loose coupling (1)

Preliminary calculations: Let $j = 0$

$$\mathbf{u}_1^{n+1(j)} = 2\mathbf{u}_1^n - \mathbf{u}_1^{n-1}$$

Step 1 (Predict):

$$\dot{\mathbf{x}}_1 = \mathbf{X}_1\left(t, \mathbf{x}_1(t), \mathbf{L}\left(\alpha, \mathbf{u}_1^n, \mathbf{u}_1^{n+1(j)}\right)\right) \xrightarrow{\text{RK4}} \mathbf{x}_1^{n+1(j)}$$

$$\mathbf{y}_1^{n+1(j)} = \mathbf{Y}_1\left(t^{n+1}, \mathbf{x}_1^{n+1(j)}, \mathbf{L}\left(\alpha, \mathbf{u}_1^n, \mathbf{u}_1^{n+1(j)}\right)\right)$$

$$\mathbf{u}_2^{n+1(j)} = \mathbf{y}_1^{n+1(j)}$$

Step 2 (Substitute & predict):

$$\dot{\mathbf{x}}_2 = \mathbf{X}_2\left(t, \mathbf{x}_2(t), \mathbf{L}\left(\alpha, \mathbf{u}_2^n, \mathbf{u}_2^{n+1(j)}\right)\right) \xrightarrow{\text{RK4}} \mathbf{x}_2^{n+1(j+1)}$$

$$\mathbf{y}_2^{n+1(j+1)} = \mathbf{Y}_2\left(t^{n+1}, \mathbf{x}_2^{n+1(j+1)}, \mathbf{L}\left(\alpha, \mathbf{u}_2^n, \mathbf{u}_2^{n+1(j)}\right)\right)$$

$$\mathbf{u}_1^{n+1(j+1)} = \mathbf{y}_2^{n+1(j+1)}$$

Predictor-Corrector Loose coupling (2)

Step 3 (Substitue & correct):

$$\dot{\mathbf{x}}_1 = \mathbf{X}_1\left(t, \mathbf{x}_1(t), \mathbf{L}\left(\alpha, \mathbf{u}_1^n, \mathbf{u}_1^{n+1(j+1)}\right)\right) \xrightarrow{\text{RK4}} \mathbf{x}_1^{n+1(j+1)}$$

$$\mathbf{y}_1^{n+1(j+1)} = \mathbf{Y}_1\left(t^{n+1}, \mathbf{x}_1^{n+1(j+1)}, \mathbf{L}\left(\alpha, \mathbf{u}_1^n, \mathbf{u}_1^{n+1(j+1)}\right)\right)$$

$$\mathbf{u}_2^{n+1(j+1)} = \mathbf{y}_1^{n+1(j+1)}$$

Stop or Repeat: Let $(j+1) \rightarrow j$. If $j = j_{max}$, let

$$\mathbf{x}_1^{n+1} = \mathbf{x}_1^{n+1(j_{max})}, \qquad \mathbf{x}_2^{n+1} = \mathbf{x}_2^{n+1(j_{max})},$$

$$\mathbf{u}_1^{n+1} = \mathbf{u}_1^{n+1(j_{max})}, \qquad \mathbf{u}_2^{n+1} = \mathbf{u}_2^{n+1(j_{max})}$$

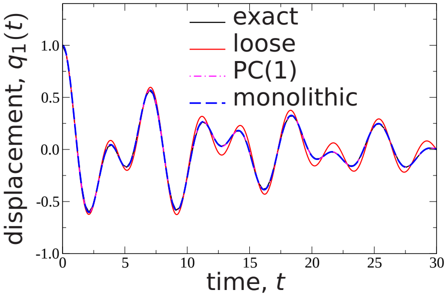
and proceed to next time step; otherwise, repeat Steps 2 and 3

We denote this approach PC(j_{max})

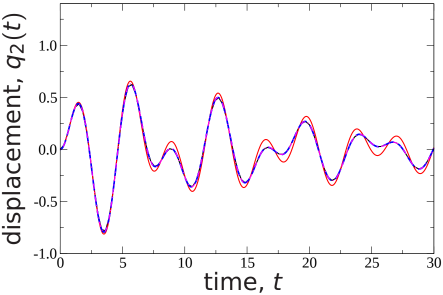
Preliminary results: Histories

Examine loose coupling where each system is time integrated with RK4 and $\Delta t = 0.05$

System 1			System 2				
m_1	c_1	k_1	m_2	c_2	k_2	c_c	k_c
1.0	0.1	1.0	1.0	0.1	1.0	0.01	1.0



(a) System 1

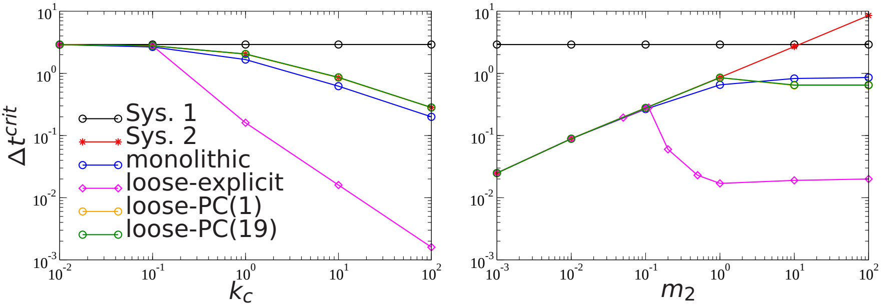


(b) System 2

Exact, PC(1), and monolithic histories are indistinguishable

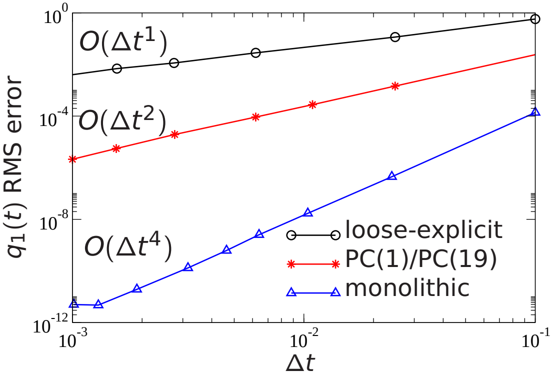
Preliminary results: Numerical stability

- Vary k_c and m_2, independently, and examine effect on critical time increment, Δt^{crit}
- "Sys. 1" and "Sys. 2" data are for uncoupled time integration
- Loose coupling significantly degrades numerical stability
- PC coupling shows similar stability w.r.t. mono. treatment



Preliminary results: Numerical accuracy

- Examine convergence rates for RK4 time integration; consider RMS error of $q_1(t)$ over $0 < t \leq 30$



- Monolithic system shows fourth-order convergence
- Loose explicit coupling is only first-order accurate
- PC(1)/PC(19) stong loose coupling is only second-order accurate

Future work

- Examine loose coupling with a multi-step method like Adams-Bashforth-Moulton
  - Pursue coupling scheme that retains maximum accuracy
- Extend example-problem set to include
  - discrete-time partition
  - nonlinearity
  - partition-internal contraints
- Compare loose and tight coupling in terms of accuracy vs. computational cost
- Work to be presented at the AIAA *51st Aerospace Sciences Meeting* [4]

References

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